



One or infinite optimal city sizes? In search of an equilibrium size for cities

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1. Aim of the paper

- To enter the debate on optimal city size by providing a theoretical model that overcomes the existing paradoxes present in the literature,
- To test the model on a European city sample
- Policy implications



2. Existing paradoxes and novelty of the approach 1/3

- Alonso (1971) highlights two provocative but opposite statements in explaining the optimal dimension of cities:
 - one single optimal city size should exist (*“how big is too big?”*)
 - optimality *“will vary from city to city, from society to society”*
- Richardson (1972) confirmed a “skeptic’s view”, underlining that an evident paradox existed between the theoretical acceptance of an “optimal city size” and the contradictory structure of urban systems in the real world.
- In the revised Central Place Model (Beckmann, McPherson, 1970), according to the different functional mix present in each urban rank, higher rank cities are expected to show a wider size, while cities belonging to the same rank share the same size.



2. Existing paradoxes and novelty of the approach 2/3

- Given the unsettled paradox, for a long time scientific efforts were redirected outside the problem of searching for an 'optimal' size and mainly dedicated to the identification of urban specificities that affect urban costs and benefits.
- In this paper an **intermediate position** is assumed between the idea of a single, 'optimal' size for any city and that of an infinite plurality of 'optimal', but unexplained sizes:
 - ***cities are assumed to be comparable***, sharing common costs and benefits functions, therefore allowing cross-sectional empirical analyses and considering other determinants of urban benefits and costs beyond pure city size;
 - *on the other hand, each city maintains its own specificity and uniqueness*, and consequently is attributed its own 'equilibrium' size in an econometric model equating marginal costs and benefits to urban size.
- Therefore the possibility of devising policy strategies for urban growth or containment is saved



3. Previous literature on determinants of urban size 1/2

The determinants of urban size - Conventional approaches

- *Indivisibilities and productivity*: (Segal, 1976, Rousseaux and Proud'homme, 1992; Rousseaux, 1995)
- *Urban diversity as source of creativity*: Chinitz (1961) and Jacobs (1969)
- *Agglomeration as a facilitator of social interaction*: Martin et al. (2011)
- *Agglomeration and Human Capital*: Glaeser and Mare (2010)
- *Human capital and local synergies as sources of learning*: human capital and “tacit knowledge” (Polanyi, 1966; Bathelt et al., 2004); shared values, common codes of behaviour, sense of belonging and mutual trust (cities as *innovative milieux*, Camagni 1991, 1999)
- *Amenities as sources of urban attractiveness*: Rappaport, 2007; Cheshire and Magrini, 2006
- *Environmental costs and social conflicts*: (Ridker and Henning, 1967; Wilkinson, 1973; Freeman, 1971; Getz and Huang, 1978; Izraeli, 1987)



3. Previous literature on determinants of urban size 2/2

The determinants of urban size - Unconventional approaches

- *Urban functions and urban ranks* (SOUDY: Camagni et al., 1986)
- *City networks: “complementarity networks”* (spatial division of labour) and *“synergy networks”* (among cities performing similar functions): Camagni (1993)
- *Urban form and sprawl*: compactness is efficient and sustainable (Camagni et al., 2002).



4. The model 1/3

We start by assuming the following implicit urban cost and urban benefit functions:

1. $C = f(\textit{size}, \textit{rent}, \textit{sprawl}, \textit{malaise})$
2. $B = f(\textit{size}, \textit{amenities}, \textit{diversity}, \textit{density}, \textit{functions}, \textit{networks})$

We next assume a Cobb-Douglas form for the two functions above:

3. $C = \textit{size}^\alpha \textit{rent}^\beta \textit{malaise}^\delta \textit{sprawl}^\gamma$
4. $B = \textit{size}^\kappa \textit{amenities}^\zeta \textit{diversity}^\vartheta \textit{density}^\chi \textit{functions}^\mu \textit{networks}^\nu$



4. The model 2/3

Assuming spatial equilibrium across space holds, we equalize marginal costs and marginal benefits to city size:

$$\begin{aligned}\frac{\partial C}{\partial \text{size}} &= \frac{\partial B}{\partial \text{size}} \Rightarrow \alpha \text{size}^{\alpha-1} \text{rent}^{\beta} \text{malaise}^{\delta} \text{sprawl}^{\gamma} = \\ &= \kappa \text{size}^{\kappa-1} \text{amenities}^{\zeta} \text{diversity}^{\vartheta} \text{density}^{\chi} \text{functions}^{\mu} \text{networks}^{\nu}\end{aligned}$$

Solving for size gives:

$$\frac{\text{size}^{\alpha-1}}{\text{size}^{\kappa-1}} = \frac{\kappa \text{amenities}^{\zeta} \text{diversity}^{\vartheta} \text{density}^{\chi} \text{functions}^{\mu} \text{networks}^{\nu}}{\alpha \text{rent}^{\beta} \text{malaise}^{\delta} \text{sprawl}^{\gamma}}$$

That is:

$$\text{size}^{\alpha-\kappa} = \frac{\kappa \text{amenities}^{\zeta} \text{diversity}^{\vartheta} \text{density}^{\chi} \text{functions}^{\mu} \text{networks}^{\nu}}{\alpha \text{rent}^{\beta} \text{malaise}^{\delta} \text{sprawl}^{\gamma}}$$



4. The model 3/3

Re-arranging terms in the log-linear form, we obtain:

$$\begin{aligned} \ln(\text{size}) = & \frac{\ln(\kappa/\alpha)}{(\alpha - \kappa)} + \frac{\zeta}{(\alpha - \kappa)} \ln(\text{amenities}) + \frac{\vartheta}{(\alpha - \kappa)} \ln(\text{diversity}) + \\ & + \frac{\chi}{(\alpha - \kappa)} \ln(\text{density}) + \frac{\mu}{(\alpha - \kappa)} \ln(\text{functions}) + \frac{\nu}{(\alpha - \kappa)} \ln(\text{networks}) + \\ & - \frac{\beta}{(\alpha - \kappa)} \ln(\text{rent}) - \frac{\delta}{(\alpha - \kappa)} \ln(\text{malaise}) - \frac{\gamma}{(\alpha - \kappa)} \ln(\text{sprawl}) \end{aligned}$$

This equation is the basis of our analysis.



5. The data set for the European sample

Type of variable	Class of variable	Variable	Measure	Years	Source of raw data
Dependent	Physical size of cities	Size	Population levels in 59 LUZ (1)	Average 2004-2006	ESPON/Urban Audit
	<i>Traditional urban benefits</i>				
	Quality of life	Amenities	Tourist inflows over available years	Average 2001-2004	Urban Audit
	Urban creativity	Diversity	Sectoral diversity index measured as 1 - the share of top 5 NACE 2 digits industries (2)	1990	ESPON
	Agglomeration economies	Density	Population density	Average 1989-2003	Urban Audit
Independent	<i>Traditional urban costs</i>				
	Cost of the city	Rent	Cost of average quality apartment per square meter	Average 1991-2004	Various (see Appendix 2)
	Social conflict	Malaise	Number of crimes per 1,000 population per year	Average 1989-2003	Urban Audit
	<i>Nonconventional urban benefits</i>				
	City networks	Networks	Number of participations in Framework Programme 5 projects over labour force	Average 1998-2002	CORDIS
High level urban functions	Functions	Workforce in ISCO professions 1 and 2 (respectively, legislators, senior officials and managers and professionals) over total FUA labour force (2)	Average 2002-2004	ESPON	
<i>Nonconventional urban costs</i>					
Diffused urban form	Sprawl	Percentage of non-built-up area of the total area of FUA. Built-up areas include artificial areas according to the CORINE Land Cover nomenclature.	1990	ESPON	



5. The European sample





6. Empirical results: European cities 1/3

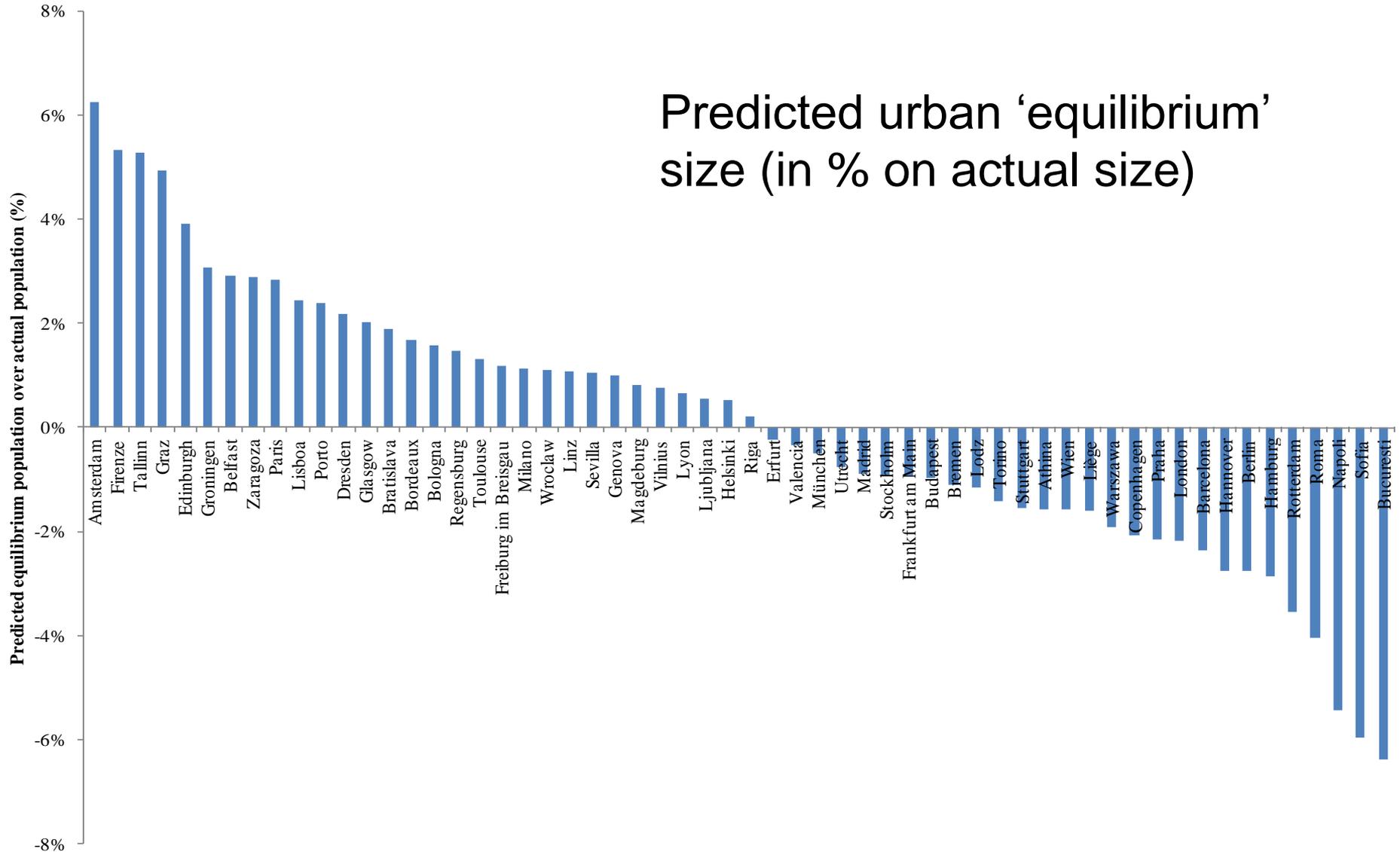
Dependent variable: equilibrium city population	(1)	(2)	(3)	(4)	(5)	(6)
Constant	8.80*** (1.49)	12.54*** (1.57)	11.05*** (1.49)	3.93*** (2.70)	1.58 (2.29)	9.93*** (2.01)
Land rent	0.70*** (0.20)	0.43** (0.18)	0.36** (0.17)	-0.12 (0.15)	-0.15 (0.12)	-0.35** (0.14)
Malaise	-	-0.16* (0.09)	-0.16** (0.08)	-0.12* (0.06)	-0.11** (0.05)	-0.10* (0.05)
Urban amenities	-	-	-	0.47*** (0.07)	0.43*** (0.07)	0.32*** (0.07)
Urban diversity	-	-	-	1.69** (0.68)	2.05*** (0.57)	0.83* (0.46)
Density	-	-	0.27*** (0.10)	-	0.26*** (0.07)	-
City networks	-	-	-	-	-	0.12** (0.05)
Urban functions	-	-	-	-	-	0.20** (0.09)
Dummy small countries	-	-	-	-	-	-0.25* (0.13)
Dummy financial capital	-	-	-	-	-	0.60*** (0.17)
Sprawl	-	-0.37*** (0.10)	-0.20** (0.09)	-0.29*** (0.07)	-0.21*** (0.08)	-0.30*** (0.08)
R ²	0.20	0.39	0.45	0.70	0.75	0.78
Joint F test	12.51***	13.31***	12.73***	37.56***	32.67***	21.01***
Robust standard errors	Yes	Yes	Yes	Yes	Yes	Yes
Number of observations	59	59	59	59	59	59

Notes: Dependent variable: Equilibrium city size (Log city population 2004-2006). Standard errors in parentheses.

***, **, * imply significance at 10, 5 and one percent, respectively.



6. Empirical results: European cities 2/3



6. Empirical results: the I.V. Model 3/3

Dependent variable: equilibrium city population	(1)	(2)	(3)
Constant	9.93*** (2.01)	12.79*** (3.17)	12.78*** (2.48)
Land rent	-0.35** (0.14)	-0.32** (0.13)	-0.41*** (0.17)
Malaise	-0.10* (0.05)	-0.24* (0.14)	-0.20*** (0.07)
Urban amenities	0.32*** (0.07)	0.29*** (0.07)	0.21*** (0.07)
Urban diversity	0.83* (0.46)	0.88** (0.45)	0.80** (0.39)
City networks	0.12** (0.05)	0.10** (0.05)	0.13** (0.06)
Urban functions	0.20** (0.09)	0.13 (0.08)	0.46* (0.25)
Dummy small countries	-0.25* (0.13)	-0.22* (0.12)	-0.17 (0.16)
Dummy financial capital	0.60*** (0.17)	0.46* (0.24)	0.62** (0.23)
Sprawl	-0.30*** (0.08)	-0.27*** (0.07)	-0.22** (0.09)
R ²	0.78	0.82	0.75
Joint F test	21.01***	22.97***	37.40***
Number of observations	59	59	59
Variable instrumented	-	Malaise	Functions
Instruments used	-	Social capital indicators, time-lagged per capita FUA wealth.	Social capital indicators, indicators of a culturally advanced society, presence in the FUA of at least one university among the top 500 in the 2003 Shanghai ranking.
Partial R-squared of excluded instruments	-	0.13	0.14
Cragg-Donald statistic	-	9.08** (0.02)	9.14** (0.04)
Anderson corr. LR statistic (identification/IV relevance test)	-	8.45** (0.04)	8.50* (0.08)
C-statistic for exogeneity	-	5.30* (0.07)	6.43** (0.04)

Notes: Dependent variable: Equilibrium city size (Log city population 2004-2006). Standard errors in parentheses. * = ***, **, * imply significance at 10, 5 and one percent, respectively.



7. Conclusions

- An intermediate theoretical position is taken between “one single optimal size” and “infinite sizes”, identifying an “equilibrium” size for each city according to a series of characteristics - both traditional and more recently proposed - impacting on urban costs and benefits.
- The evidence suggests that recent conceptual paradigms explain much of current disparities in terms of urban performance and urban size.
- While rent represents the single highest cost associated to urban size, cities now benefit not only from attracting highly educated professionals, and hosting a rich and diversified labour market, but also from pure amenities and compact urban form.
- Being connected to a network also fosters urban performance.
- The residual between predicted and actual size for each city may be interpreted as: a) effect of our ignorance on other determinants of urban size; b) effect of good/bad urban governance; c) growth/decline potential
- Planning and urban policies matter, when smartly integrated with a sound economic strategy concerning urban functions, diversity and networking!



And for your attention,

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